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Connections on Discrete Fibre Bundles

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Abstract. A new approach to gauge fields on a discrete space-time is proposed, in which the fundamental object is a discrete version of a principal fibre bundle. If the bundle is twisted, the gauge fields are topologically non-trivial automatically.

1. Introduction

For both computational and conceptual purposes it is very often convenient to formulate physical theories on a space or space-time lattice. A lattice discretizes space but preserves some of its topological structure through the provision of links between neighbours of the lattice. A smooth curve is approximated not by an arbitrary set of lattice points but by points joined by these links and this allows one to determine, for example, whether a closed loop in a 3-dimensional lattice is knotted or not. By a lattice, one usually means an infinite set of points in $\mathbb{R}^n$ generated by a discrete translation group. We shall actually be interested in compact, spherical spaces and shall use the word lattice to mean a finite set of points on the sphere generated by a discrete subgroup of the rotation group.

In this paper, we propose a new way to consider lattice gauge fields. Existing treatments are not really satisfactory because the topological properties of continuum gauge fields are easily lost in the discretization of space. This leads to difficulties with fermions on the lattice; it seems impossible to incorporate chiral fermions, and the anomalies of the continuum theory disappear.

Lüscher [1] has proposed a definition of the topological charge of a lattice gauge field in four dimensions, which has been developed and implemented numerically by Woit [2], Phillips and Stone [3], Teper [4] and others. A characteristic feature of Lüscher's idea is that, locally, in a suitable gauge, the group elements on the links of the lattice (Wilson link variables) which define the lattice gauge field are close to the identity. However, there are difficulties. The definition fails for some exceptional configurations, and it is algebraically rather complicated. Also, the underlying fibre bundle is not fundamental in Lüscher's