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The Internal Symmetry Group of a Connection on a Principal Fiber Bundle with Applications to Gauge Field Theories

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Abstract. The internal symmetry group of a connection on a principal fiber bundle P is studied. It is shown that this group is a smooth proper Lie transformation group of P, which, if P is connected, is also free. Moreover, this group is shown to be isomorphic to the centralizer of the holonomy group of the connection. Several examples and applications of these results to gauge field theories are given.

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Introduction

Let $P(M, G)$ denote a principal fiber bundle with structure group G over a connected manifold M, let $\mathrm{AUT}(P)$ denote the group of automorphisms of P, and let $\mathrm{Aut}(P)$ denote the normal subgroup of automorphisms of P that cover the identity diffeomorphism of M. Let ω be the connection 1-form of a connection on P. If $F \in \mathrm{AUT}(P)$, let $F^*\omega$ denote the pullback of ω by F. Then

$$\mathrm{AUT}_\omega(P) = \{F \in \mathrm{AUT}(P) \mid F^*\omega = \omega\}$$

is the *symmetry group of* ω, and

$$I_\omega(P) = \{F \in \mathrm{Aut}(P) \mid F^*\omega = \omega\} = \mathrm{Aut}(P) \cap \mathrm{AUT}_\omega(P)$$