Commun. Math. Phys. 113, 191–208 (1987)

Communications in Mathematical Physics

Absence of Shocks in an Initially Dilute Collisionless Plasma*

Robert T. Glassey[1] and Walter A. Strauss[2]

[1] Department of Mathematics, Indiana University, Bloomington, IN 47405, USA
[2] Department of Mathematics, Brown University, Providence, RI 02912, USA

Abstract. The Cauchy Problem for the relativistic Vlasov–Maxwell equations is studied in three space dimensions. It is assumed that the initial data satisfy the required constraints and have compact support. If in addition the data have sufficiently small C^2 norm, then a unique C^1 solution to this system is shown to exist on all of spacetime.

1. Introduction

A plasma is said to be collisionless if the collisions between the particles are unimportant relative to the electromagnetic forces. Such a plasma is governed by the Vlasov–Maxwell equations. An ionized gas which is sufficiently hot, such as a powered-up fusion reactor, or sufficiently dilute, such as the solar wind, the ionosphere or a collection of galactic nebulae, is thought to be well modeled by these equations. In this paper we prove that if a plasma is sufficiently dilute initially, then it remains so for all time and no shock or other singularity can ever develop.

We postulate several (N) species of particles (electrons, different types of ions, etc.) with masses m_α, charges e_α and densities $f_\alpha(t,x,v)$ ($t \geqq 0$, $x \in \mathbb{R}^3$, $v \in \mathbb{R}^3$, $1 \leqq \alpha \leqq N$). The total charge and current densities are

$$\rho(t,x) = 4\pi \int_{\mathbb{R}^3} \sum_\alpha e_\alpha f_\alpha \, dv, \tag{1}$$

$$j(t,x) = 4\pi \int_{\mathbb{R}^3} \sum_\alpha e_\alpha \hat{v}_\alpha f_\alpha \, dv, \tag{2}$$

where, in the relativistic version we consider,

$$\hat{v}_\alpha \equiv v[m_\alpha^2 + |v|^2/c^2]^{-1/2} \tag{3}$$

has the interpretation of velocity, v is the momentum, and c is the speed of light. Each f_α will satisfy a continuity equation (the Vlasov equation) which is coupled

* Research supported in part by NSF DMS 85-20662 and NSF DMS 84-20957