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Dual Field Formulation of Quantum Integrable Models

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Abstract. Equal time correlators are studied in completely integrable models. The main example is the quantum non-linear Schrödinger equation. Introduction of an auxiliary Fock space permits us to represent the generating functional of correlators in the form of a determinant of the integral operator.

1. Introduction

The Algebraic Bethe Ansatz [1] permits us to advance in the analysis of completely integrable quantum models. Correlation functions are of special interest. The approach of papers [2–6] will be developed here. We shall illustrate new results in correlation functions on the example of the quantum non-linear Schrödinger equation (another name of the model is the one-dimensional Bose gas). The Hamiltonian H and commutation relations of the model are:

$$H=\int_0^L dx(\partial_x\psi^+\partial_x\psi+c\psi^+\psi^+\psi\psi)\,,\qquad c>0\,,$$
$$[\psi(x),\psi^+(y)]=\delta(x-y)\,,\qquad [\psi(x),\psi(y)]=[\psi^+(x),\psi^+(y)]=0\,. \tag{1.1}$$

Here ψ is a canonical Bose field, L is the length of periodic box and c is a coupling constant. The Fock space in which the quantum fields ψ,ψ^+ act will be called the initial Fock space, because later there will appear another auxiliary Fock space. The algebraic vacuum $|0\rangle$ in the initial Fock space is defined in the standard way:

$$\psi(x)|0\rangle=0\,. \tag{1.2}$$

The dual algebraic vacuum $\langle 0|=|0\rangle^+$ is also important

$$\langle 0|\psi^+(x)=0\,,\qquad \langle 0|0\rangle=1\,. \tag{1.3}$$

We shall consider the ground state of the model in the sector with fixed density D and calculate correlators for this case. The correlators of conserving currents are of main interest for us. The simplest example is:

$$\langle\psi^+(x)\psi(x)\psi^+(0)\psi(0)\rangle\,,\qquad x>0\,. \tag{1.4}$$