Commun. Math. Phys. 113, 137–154 (1987)

Communications in
Mathematical
Physics

Marguils Distributions for Anosov Flows

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Abstract. For the strong unstable foliation (or horocycle foliation) of an Anosov flow there exists a unique transverse measure called the Margulis measure. In this paper we extend Margulis' results to more general "transverse distributions" for the foliation. As an application we derive our main result: The non-zero analytic extension to a strip of the Selberg zeta function for compact surfaces of constant negative curvature persists under small perturbations in the metric. There is an equivalent formulation in terms of the Fourier transform of the correlation function.

0. Introduction

In the study of Axiom A flows there are two functions (of a single complex variable) which contain a considerable amount of information on the dynamics of the flow – the zeta function and the Fourier transform of the correlation function (which describes the way in which the flow mixes). Their meromorphic domains influence the asymptotic growth of closed orbit periods (through the zeta function [14]) and the rate of mixing of the flow (through the correlation function [16, 21]).

David Ruelle has gone some way in analysing the residues which occur for the Fourier transform and he has given them some interpretation in terms of "Gibbs distributions" [22] (similar in nature to distributions in the Schwartzian sense, but which are complex valued). However, this does not broach the problem of locating the poles. There is a simple correspondence between poles for the zeta function and poles for the Fourier transform of the correlation function (within appropriate domains) which reduces this to a single problem [16].

In this article we give an approach to determining the location of the poles and to giving them, along with the residues, a clearer interpretation, at least for Anosov flows.

We shall formulate most of our results at the level of three-dimensional Anosov flows. This will be most appropriate for our main application.

Our approach to these problems is to consider transverse distributions (generalising transverse measures) for the natural foliation associated with the