Commun. Math. Phys. 113, 115–136 (1987)

Communications in Mathematical Physics

A Conformally Invariant Limit of the Critical Lattice Ising Model

Yu. A. Bashilov and S. V. Pokrovsky*

Academy of Sciences, USSR, Space Research Institute, Profsoyuznaya 84/32, SU-17810 Moscow, USSR

Abstract. A detailed analysis of an expansion in powers of $1/N$ ($N \gg 1$) for the Hamiltonian and the transfer-matrix of the Ising chain consisting of N sites is presented. A special attention is paid to a term proportional to $1/N$ closely related to the theory of free massless Majorana fermions. An effective Hamiltonian isomorphic to that in conformally invariant theory is derived. The eigenvectors of the Ising Hamiltonian are classified in the framework of conformal algebra representation theory. The momentum and the energy for these states are expressed through the central charge and conformal dimensions. A similar relation for the logarithms of the transfer-matrix eigenvalues is ascertained. These are complex-valued functions of a spectral parameter θ. A real part of such a function is shown to be proportional to $\sin\theta$, while an imaginary one – to $\cos\theta$. A direct geometrical interpretation of the lattice spectral parameter in the context of the conformally invariant theory for fermions inhabiting a torus is indicated. In other words, these fermions are represented as analytic anticommuting variables double (anti) periodic in a complex plane. The value of the spectral parameter coincides with an angle between these (anti) periods. A general scheme for the above expansion presumably suitable for a wide class of exactly solvable models is conjectured.

1. Introduction

A bootstrap program for classification of possible types of critical behavior in statistical systems proposed by Polyakov more than ten years ago [1, 2] attracted new interest in the last few years. In the fundamental work by Belavin et al. [3], the conformal invariance in two dimensions (ultimately restrictive) was used together with bootstrap conditions to fix completely the operator algebra. Self-consistency conditions (locality plus associativity) combined with conformal invariance can be interpreted as a system of bootstrap equations. Any solution of this system represents some possible type of critical behavior. Though the problem of

* To whom all correspondence should be addressed