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On the Equivalence of the Two Most Favoured Calabi–Yau Compactifications

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Abstract. We discuss the two known multiply connected Calabi-Yau manifolds which give rise to three generations of elementary particles when chosen as the classical vacuum configuration of the $E_8 \times E_8$ heterotic superstring. It is shown that these two manifolds are diffeomorphic.

1. Introduction

In addition to providing a potentially consistent unification of the fundamental forces, superstring theory [1, 2] has the appealing aspect of leading to highly constrained low energy models of particle interactions. Once a vacuum state is chosen much of the low energy structure of such models is determined. There is, of course, a catch [3]. One must first choose a vacuum configuration. Ideally, one hopes that further investigation of the underlying theory will ultimately show that the vacuum state is determined dynamically. For now, however, one does the next best thing by combining the demands of internal consistency and phenomenological viability. Analysis of this sort from a number of viewpoints has shown [4], that this favours vacuum configurations based upon Calabi-Yau compactifications; that is, vaccua of the form $M_4 \times K$, where M_4 is four dimensional Minkowski space and K is a three complex dimensional Kähler manifold with vanishing first Chern class. This certainly narrows down the choice, but, alas, there are many Calabi-Yau manifolds to choose from [5, 6]. As noted, though, after a choice of the Calabi-Yau vacuum configuration K is made, much of the resulting low energy phenomenology may be extracted from the topological and cohomological properties of the manifold. In fact, in the case of the $E_8 \times E_8$ Heterotic string, the number of generations of elementary particle multiplets (i.e. 27's of E_6) is given by one half of the Euler characteristic of K, $|\chi(K)/2|$ (after the $SU(3)$ spin connection ω of K

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