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Realization of Holonomic Constraints and Freezing of High Frequency Degrees of Freedom in the Light of Classical Perturbation Theory. Part I

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Abstract. The so-called problem of the realization of the holonomic constraints of classical mechanics is here revisited, in the light of Nekhoroshev-like classical perturbation theory. Precisely, if constraints are physically represented by very steep potential wells, with associated high frequency transversal vibrations, then one shows that (within suitable assumptions) the vibrational energy and the energy associated to the constrained motion are separately almost constant, for a very long time scale growing exponentially with the frequency (i.e., with the rigidity of the constraint one aims to realize). This result can also be applied to microscopic physics, providing a possible entirely classical mechanism for the "freezing" of the high-frequency degrees of freedom, in terms of non-equilibrium statistical mechanics, according to some ideas expressed by Boltzmann and Jeans at the turn of the century. In this Part I we introduce the problem and prove a first theorem concerning the realization of a single constraint (within a system of any number of degrees of freedom). The problem of the realization of many constraints will be considered in a forthcoming Part II.

1. Introduction

1.1 In this paper, and in a forthcoming second part, we will be concerned with Hamiltonian dynamical systems of the form

$$H_\omega(p,x,\pi,\xi) = h_\omega(\pi,\xi) + \hat{h}(p,x) + f(p,x,\pi,\xi), \tag{1.1}$$

where $h_\omega(\pi,\xi)$, with $(\pi,\xi) = (\pi_1,\ldots,\pi_\nu,\xi_1,\ldots,\xi_\nu)\in\mathbf{R}^{2\nu}$, is the Hamiltonian of a set of ν uncoupled harmonic oscillators of angular frequency $\omega = (\omega_1,\ldots,\omega_\nu)$, i.e.

$$h_\omega(\pi,\xi) = \frac{1}{2}\sum_{i=1}^{\nu}(\pi_i^2 + \omega_i^2\xi_i^2), \tag{1.2}$$