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# Global Validity of the Boltzmann Equation for a Three-Dimensional Rare Gas in Vacuum

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**Abstract.** We consider a system of $N$ hard spheres in the Boltzmann–Grad limit (i.e. $d \to 0, N \to \infty, Nd^2 \to \lambda^{-1} > 0$, where $d$ is the diameter of the spheres). If $\lambda$ is sufficiently large, and if the joint distribution densities factorize at time zero, with the one particle distribution decaying sufficiently rapidly in space and velocities, we prove that the time evolved one-particle distribution converges for all times to the solution of the Boltzmann equation with the same initial datum. This result improves and is based on a previous paper [1], valid only in two dimensions.

**1.** In a recent paper [1] the validity of the Boltzmann equation has been proved for a cloud of gas of hard spheres in the two-dimensional vacuum. In the present paper we extend this result to the more physically relevant three-dimensional case.

A part of the techniques necessary to obtain the present result are contained in [1] to which we address the reader for motivations, general comments, further references and notation. We briefly review the result of [1] and explain why that approach fails in dimension three. Following the same notation, we denote by $\Gamma_N$ the phase space of a system of $N$ spheres of diameter $d$ in $\mathbb{R}^\nu (\nu = 2, 3)$, $X = \{x_1 v_1 \dots x_N v_N\}$, $x_i v_i \in \mathbb{R}^\nu \times \mathbb{R}^\nu$ a phase point, $\phi_t^d$ the (almost everywhere defined) time evolution of the system, (for which $\phi_t^d(X) = \{x_1(t) v_1(t) \dots x_N(t) v_N(t)\}$ is the trajectory of the phase point $X$), $\mu^d(dX) = \mu^d(X)\, dX$ an absolutely continuous, symmetric (in the exchange of particles), probability measure on $\Gamma_N$ at time zero, $\mu_t^d(X)$ the time evolved density, $f_{k,t}^d\, t \geqq 0, 0 < k \leqq N$ the joint distribution densities. Finally $S^d(t)$ is defined by:

$$S^d(t) f_k^d(x_1 v_1 \dots x_k v_k) = f_k^d(\phi_{-t}^d(\{x_1 v_1 \dots x_k v_k\})). \tag{1.1}$$

The following equation is satisfied by the family $f_{k,t}^d$:

$$f_{k,t}^d = S^d(t) f_k^d + \int_0^t ds\, S^d(t-s) C_{k,k+1}^d f_{k+1,s}^d, \tag{1.2}$$

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