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Hyperscaling Inequalities for Percolation

Hal Tasaki*

Physics Department, Princeton University, PO Box 708, Princeton, NJ 08544, USA

Abstract. A set of critical exponent inequalities for independent percolation which saturate under the hyperscaling hypothesis is proved. One of the consequences of the inequalities is the lower bound $d_C \geqq 6$ for the upper critical dimension. The proof is based on a rigorous version of the finite size scaling argument which extends easily to other systems such as Ising ferromagnets.

1. Introduction

In the present paper, we prove the following critical exponent inequalities for the independent percolation [1]

$$(d-2+\eta)\delta_r \geqq 2 \ , \tag{1.1}$$

$$(d-2+\eta)\nu' \geqq 2\beta \ , \tag{1.2}$$

$$d\nu' \geqq \gamma' + 2\beta \ , \quad d\nu_{\max} \geqq \gamma + 2\beta \ , \tag{1.3}$$

$$d\nu \geqq 2\Delta - \gamma \ , \tag{1.4}$$

$$d\nu' \geqq \Delta' + \beta \ , \quad d\nu_{\max} \geqq \Delta + \beta \ , \tag{1.5}$$

$$(d-2+\eta)\mu\delta \geqq 2 \ , \tag{1.6}$$

$$d\mu \geqq 1 + 1/\delta \ . \tag{1.7}$$

These inequalities are of particular interest because of their close relation to the so-called hyperscaling hypothesis. If the hyperscaling hypothesis is valid, all the inequalities (1.1)–(1.7) become exact equalities.

Usually it is believed that the hyperscaling relations hold only in sufficiently low dimensions. As for independent percolation in two dimensions, Kesten [2] has recently proved almost all of the expected hyperscaling relations. However the validity of the hyperscaling hypothesis in dimensions higher than two is still wide

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