Commun. Math. Phys. 113, 27–48 (1987)

Communications in
Mathematical
Physics

On the Upper Critical Dimension of Bernoulli Percolation

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Abstract. We derive a set of inequalities for the d-dimensional independent percolation problem. Assuming the existence of critical exponents, these inequalities imply:

$$f+\nu \geqq 1+\beta_Q,$$

$$\mu+\nu \geqq 1+\beta_Q,$$

$$\zeta \geqq \min\left\{1, \frac{\nu'}{\nu}\right\},$$

where the above exponents are f: the flow constant exponent, $\nu(\nu')$: the correlation length exponent below (above) threshold, μ: the surface tension exponent, β_Q: the backbone density exponent and ζ: the chemical distance exponent. Note that all of these inequalities are mean-field bounds, and that they relate the exponent ν defined from below the percolation threshold to exponents defined from above threshold. Furthermore, we combine the strategy of the proofs of these inequalities with notions of finite-size scaling to derive:

$$\max\{d\nu, d\nu'\} \geqq 1+\beta_Q,$$

where d is the lattice dimension. Since $\beta_Q \geqq 2\beta$, where β is the percolation density exponent, the final bound implies that, below six dimensions, the standard order parameter and correlation length exponents cannot simultaneously assume their mean-field values; hence an implicit bound on the upper critical dimension: $d_c \geqq 6$.

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* Work supported by the U.S. Army Research Office through the Mathematical Sciences Institute of Cornell University

** Work supported by the DOE under Grant No. DE-AC01-83ER13044

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