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String Quantization on Group Manifolds and the Holomorphic Geometry of Diff S^1/S^1 ⋆

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Abstract. The recent results by Bowick and Rajeev on the relation of the geometry of Diff S^1/S^1 and string quantization in $\mathbb{R}^{d,1}$ are extended to a string moving on a group manifold. A new derivation of the curvature formula $(-\frac{26}{12}m^3+\frac{1}{6}m)\delta_{n,-m}$ for the canonical holomorphic line bundle over Diff S^1/S^1 is given which clarifies the relation of that bundle with the complex line bundles over infinite-dimensional Grassmannians, studied by Pressley and Segal.

I. Introduction

Recently Frenkel, Garland and Zuckerman have formulated the conditions for the consistency of string theory in the flat background $\mathbb{R}^{d,1}$ as conditions for Lie algebra cohomology for the Virasoro algebra, with coefficients in the Fock space of the string, [FGZ]. The results of Bowick and Rajeev in the Kähler geometry of the complexified tangent bundle of Diff S^1/S^1 can be seen as a step toward globalizing the algebraic approach in [FGZ], i.e. replacing Lie algebra cohomology by group cohomology. In this paper we shall carry out the program of [BR] in the case of a string on a group manifold.

Let G be a simple compact Lie group and LG the space of smooth loops in G, which is a group under point-wise multiplication of maps $S^1 \to G$. In string theory, the space LG can be considered either as the configuration space of a closed string moving in the manifold G or as the phase space of an open string. Namely, let $g(\tau, \sigma)$ be an open string parametrized by the time $\tau \in \mathbb{R}$ and the string coordinate $\sigma \in [0, \pi]$ with the boundary conditions $g'(\tau, 0) = g'(\tau, \pi) = 0$; here $g' = \frac{dg}{d\sigma}$ and $\dot{g} = \frac{dg}{d\tau}$. One can then introduce a new coordinate $h(\tau, \sigma)$ by

$$h(\tau,\sigma)=\exp[(g^{-1}\dot{g})(\tau,\sigma)+(g^{-1}g')(\tau,\sigma)], \quad 0 \leqq \sigma \leqq \pi$$
$$h(\tau,\sigma)=\exp[(g^{-1}\dot{g})(\tau,-\sigma)-(g^{-1}g')(\tau,-\sigma)], \quad -\pi \leqq \sigma \leqq 0.$$

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