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A Direct Method for Deriving a Multi-Soliton Solution for the Problem of Interaction of Waves on the x, y Plane

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Abstract. Explicit expressions are found for a multi-soliton solution of the system of equations describing the interaction of waves on the x, y plane. The proof of all necessary statements follows from the theory of matrices and is not based on the inverse scattering method. The obtained results are closely related to some problems of mathematical physics.

In the present paper we obtained explicit expressions for a multi-soliton solution of the system of equations

$$3\frac{\partial^2 u}{\partial y^2}-\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\left(3u^2+\frac{\partial^2 u}{\partial x^2}+8\kappa|\varphi|^2\right)\right]=0,\quad i\frac{\partial\varphi}{\partial y}=u\varphi+\frac{\partial^2\varphi}{\partial x^2},\tag{1}$$

describing (in a certain approximation) the interaction of a long wave with a short-wave packet propagating on the x, y plane at an angle to each other [1, 2]. Here u is the long wave amplitude, φ is the complex short-wave envelope and the parameter κ satisfies the condition $\kappa^2=1$. Though this solution was derived first by using the ideas underlying the inverse scattering method, our proofs here are based only on some very simple facts related to matrices of a very special form and have no relation to the afore-mentioned method. This is achieved in the following way.

1. Solution of an Auxiliary System of Equations

Let B be the square matrix of order $r_0=r_1+2r_2$, $r_1>0$, $r_2>0$, with the elements $B_{r,s}$, $r,s=1,\dots,r_0$. Assume that nonzero elements of the matrix B have the form

$$B_{r,s}=\begin{cases}\dfrac{f_r\exp[(\omega_r-\sigma_s)x-4(\omega_r^3-\sigma_r^3)y]}{\omega_r-\sigma_s},\\ \quad\text{if}\quad r=1,\dots,r_1,r_1+r_2+1,\dots,r_0\quad\text{and}\quad s=1,\dots,r_1+r_2,\\ -\dfrac{f_r\exp[-4(\omega_r^3-\sigma_r^3)y]}{\omega_r^3-\sigma_s^3},\quad\text{if}\quad r_1<r\leqq r_1+r_2<s\leqq r_0.\end{cases}\tag{1.1}$$