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# Superalgebras, Symplectic Bosons and the Sugawara Construction

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**Abstract.** New representations of affine Lie algebras are constructed using symplectic bosons of the sort that occur naturally in the BRST treatment of fermionic string theories. These representations are shown to have analogous properties to the current algebra representations in terms of free fermion fields, though they do not act in a positive space. In particular, the condition for the Sugawara construction of the Virasoro algebra to equal the free one is the existence of a superalgebra with a quadratic Casimir operator, paralleling the symmetric space theorem for fermionic field constructions. Both results are seen to be particular cases of a more general super-symmetric space theorem, which arises from considering an affinisation of the superalgebras. These algebras are realised in terms of free fermions and symplectic bosons and lead to a super-Sugawara construction of the Virasoro algebra. The conditions for this to equal a Virasoro algebra obtained from the free fields are provided by the super-symmetric space theorem.

## 1. Introduction

The "quark model current algebra construction" provides the simplest way of obtaining representations of affine Kac-Moody algebras. (For a review see [1].) Given an orthogonal representation of a finite-dimensional compact Lie algebra, $g$,

$$[t^a, t^b] = if_c^{ab} t^c \ , \tag{1.1}$$

described by real antisymmetric matrices, $M^a(t^a \to iM^a)$, we can obtain a representation of the untwisted affine algebra, $\hat{g}$, associated with $g$, by introducing fermi fields, $\psi^j(z)$, $1 \leqq j \leqq \dim M$, defined on the unit circle $|z|=1$ in the complex plane and setting

$$T^a(z) = \frac{i}{2} M^a_{ij} \psi^i(z) \psi^j(z) \ . \tag{1.2}$$

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