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Bosonization on Higher Genus Riemann Surfaces†

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Abstract. We prove the equivalence between certain fermionic and bosonic theories in two spacetime dimensions. The theories have fields of arbitrary spin on compact surfaces with any number of handles. Global considerations require that we add new topological terms to the bosonic action. The proof that our prescription is correct relies on methods of complex algebraic geometry.

1. Introduction

Two-dimensional quantum field theory is very special. Many surprising and beautiful results turn out to be true only in two dimensions, including for example the exact solvability of certain models, the equivalence of fermionic and bosonic field theories, and so on. One way of describing the root cause for all these miracles is to note that in two dimensions the light cone is disconnected; it consists of a left moving and a right moving branch, and massless particles stay on one branch or the other[1].

This cleavage in turn comes from the fact that in two dimensions the scalar wave operator factorizes into the product of left and right moving derivatives. In euclidean space the analogous statement is

$$\nabla^2 = \bar{\partial}^\dagger \bar{\partial} \ , \tag{1.1}$$

where $\bar{\partial}$ is the Cauchy-Riemann operator. Thus in a sense we can say that $2d$ fields are special because for them complex analysis plays a key role.

In this paper we will see how complex analytic methods can extend our understanding of $2d$ fields from surfaces with the topology of the plane (or sphere)

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[1] See, e.g. the physical discussion in § V.B of [1]