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Absence of Divergences in Type II and Heterotic String Multi-Loop Amplitudes

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Abstract. A detailed analysis is given of the two main types of degeneration of Riemann surface of arbitrary genus by domain variational theory. Explicit estimates for first and third Abelian functions are given. These estimates are used to analyse the possible divergences of type II or heterotic superstring multi-loop amplitudes for the scattering of massless particles. They are all shown to be finite at arbitrary loop order.

I. Introduction

A vast mood of euphoria has swept over the theoretical high-energy physics community with the discovery of five superstring theories as putative candidates for a unified theory of the forces of nature. These are the $O(32)$ Chan-Paton model, equal to the even G-parity sector of the 1971 dual pion model for the bosonic sector [1], the $O(32)$ or $E(8) \times E(8)$ heterotic strings [2], and the chiral or non-chiral versions of the closed superstring [3]. It is supposed that these theories all give a viable quantum gravity, though their phenomonological features may single out the $E(8) \times E(8)$ heterotic string as most promising [4]. Yet the euphoric atmosphere is based only on results of finiteness of tree and one-loop amplitudes. There is almost no information on the finiteness properties of higher-loop super-string amplitudes, nor on the convergence or otherwise of the loop perturbation expansion. Indeed there are no specific evaluations of any higher loop amplitudes. However, on the basis of general arguments, using supersymmetry and/or functional methods, it has been claimed [2, 5] that all closed superstring amplitudes are finite at all loops. It is the purpose of this paper to analyse that situation.

Only heterotic and type II closed superstrings will be discussed here, and that by means of functional techniques in the light cone gauge. Closed superstring theories are chosen rather than the open case mentioned above since the former are much simpler to consider. In the case of open superstrings there are mathematical complexities arising from the fact that the corresponding world sheets Σ are open Riemann surfaces, and some difficult mathematical problems ensue (due to the continuous part of the spectrum of the Laplace-Beltrami operator). Functional