Commun. Math. Phys. 112, 335–341 (1987)

Communications in
Mathematical
Physics

On the Geometry of Dirac Determinant Bundles in Two Dimensions*

Jouko Mickelsson**

Center for Theoretical Physics, Laboratory for Nuclear Science, and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract. The gauge and diffeomorphism anomalies are used to define the determinant bundles for the left-handed Dirac operator on a two-dimensional Riemann surface. Three different moduli spaces are studied: (1) the space of vector potentials modulo gauge transformations; (2) the space of vector potentials modulo bundle automorphisms; and, (3) the space of Riemannian metrics modulo diffeomorphisms. Using the methods earlier developed for the studies of affine Kac-Moody groups, natural geometries are constructed for each of the three bundles.

The geometry of the determinant line bundle for the left-handed Dirac operator $\gamma^\mu(\nabla_\mu + P_- A_\mu)$ on a unit sphere S^2 (P_- is the projection in left-handed components of the spinor field and A_μ is a Lie algebra valued vector potential) is known to be closely related to the geometry of an affine Kac-Moody group, [M1]. In fact, the determinant bundle Det is an associated bundle to a $U(1)$ bundle P over $\mathscr{A}/\mathscr{G}$ which in turn is a pull-back of the affine group $\hat{L}G$ with respect to a certain homotopy equivalence $\mathscr{A}/\mathscr{G} \to LG$; here, $\mathscr{A}$ is the space of vector potentials, $\mathscr{G}$ is the group of gauge transformations and LG is the loop group of the gauge group G. The affine group $\hat{L}G$ is a $U(1)$ bundle over LG. The connection form describing the geometry of P (and of Det) is a pull-back of the central projection of the Maurer-Cartan form on $\hat{L}G$, [M2].

In this paper, I want to generalize the results of [M1] and [M2] to the case when S is an arbitrary compact connected oriented Riemann surface of genus $g \geqq 2$ (the case $g=1$ is left as an exercise to the reader). In addition, I shall discuss the geometry of the determinant bundle parametrized by the space $\mathscr{M}/\mathrm{Diff}\,S$, where $\mathscr{M}$ is the space of Riemannian metrics on S. The determinant bundle on $\mathscr{M}/\mathrm{Diff}\,S$ is

* This work was supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under contract # DE-AC02-76ER03069

** Permanent address: Department of Mathematics, University of Jyväskylä, Seminaarinkatu 15, SF-40100 Jyväskylä 10, Finland