Invariants for Smooth Conjugacy
of Hyperbolic Dynamical Systems, III

José Manuel Marco* and Roberto Moriyón*

Universidad Autónoma de Madrid

Abstract. We give a characterization of Anosov diffeomorphisms smoothly conjugated to a toral automorphism in dimension two in term of the Lyapunov exponents of periodic points. We also give necessary and sufficient conditions for the regularity of solutions of the vector cohomology equations associated to an Anosov flow in three dimensions. This allows us to prove a corresponding conjugation theorem.

0. Introduction

In this paper we continue the study of smooth conjugacy problems for general Anosov systems, initiated in [MM, L1]. We are concerned with two questions.
i) Given two Anosov diffeomorphisms on the 2-dimensional torus T^2, when are they C^∞ conjugated? (This question is solved in [L1] when they are close enough in the C^1 topology). ii) Give necessary and sufficient conditions for the existence of a C^∞ conjugation between vector fields on a 3-dimensional compact manifold that generate Anosov flows (no result of this kind was previously known).

A motivation for these questions comes from inverse spectral problems in riemannian geometry (see [GK]), where one tries to construct isometries by smoothly conjugating the corresponding geodesic flows. Collet, Epstein and Gallavotti, [CEG], study them in relation with integrability properties of hamiltonian systems in a general sense. On the other hand, smooth conjugation problems are interesting by themselves, and they have given rise to important and beautiful Mathematics, as in [H].

In the case of diffeomorphisms of the torus, we probe that a C^∞ Anosov diffeomorphism is C^∞ conjugated to a toral automorphism if and only if the Lyapunov exponents of all periodic orbits are the same. In the second case we show that the periods and the Lyapunov exponents of periodic orbits are a complete set of invariants for the smooth conjugacy of one-parameter families of Anosov flows to

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