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Correlations of Peaks of Gaussian Random Fields*

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Abstract. The high peaks of a Gaussian random field are studied. Asymptotic expansions, appropriate for high peak thresholds and large spatial separations, are developed for the N-point correlation functions of the number density of high peaks, in terms of the two-point correlation of the underlying Gaussian field. Similar expressions are derived for the correlations of points, not necessarily the positions of peaks, where the field exceeds a high threshold.

I. Introduction

The mathematical modeling of many physical processes often proceeds via a statistical approach. The behavior of the random field of interest, $\varepsilon(\vec{x})$, is described by a probability distribution $P[\varepsilon]$, or equivalently by the moments of the probability distribution (we assume here that all the moments exist),

$$\langle e(\vec{x}_1)\dots\varepsilon(\vec{x}_n)\rangle = \int[d\varepsilon]\varepsilon(\vec{x}_1)\dots\varepsilon(\vec{x}_n)P[\varepsilon]. \tag{1}$$

In Eq. (1) the $\vec{x}_i$ are points in a D-dimensional Euclidean space $\mathbb{R}^D$, and the integration is over the value of ε at each point in $\mathbb{R}^D$. Some applications have a random variable which depends only on time, in which case $D = 1$. This occurs, for example, in the theory of noise in electrical networks [1]. Other applications may deal with a field ε which takes values that depend on the location in "physical" space, in which case $D = 3$. One such example is the theory of the large scale structure of the universe, where ε is the mass density fluctuation field [2, 3].

The most common probability distribution encountered in practice is a Gaussian distribution, which has

$$P[\varepsilon] = \frac{1}{Z}\exp\left[-\tfrac{1}{2}\int d^Dx d^Dy \varepsilon(\vec{x})\xi^{-1}(|\vec{x}-\vec{y}|)\varepsilon(\vec{y})\right], \tag{2}$$

where Z is a constant chosen so that $P[\varepsilon]$ is normalized to unity and $\xi^{-1}(|\vec{x}-\vec{y}|)$ is

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