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# Long Range Dynamics and Broken Symmetries in Gauge Models. The Stückelberg-Kibble Model*

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**Abstract.** The general algebraic features associated to long range dynamics like the problem of removing the infrared cutoff, the definition of the algebraic dynamics and the occurrence of variables at infinity, the essential localization (seizing of the vacuum), the effective dynamics and its covariance group (dynamical symmetry group), the generalization of Goldstone's theorem and the non-trivial Goldstone spectrum, the mass/energy gap generation by the non-trivial classical motion of the variables at infinity are explicitly shown in the Kibble model as a prototype of gauge models exhibiting the Higgs phenomenon. The relation between mass generation in the Higgs phenomenon and the plasma energy gap is also discussed.

## 1. Introduction

The general properties of the dynamics of systems with long range interactions have been discussed in previous papers [1–4] with emphasis on the occurrence of variables at infinity in the time evolution of (quasi) local variables. The evidence that the above structures are indeed realized has been shown in various cases like a class of spin models [1], the BCS model [1], the Coulomb gas in uniform background [5]. From the above examples it appears that the occurrence of variables at infinity is a generic feature associated to Coulomb like interactions and therefore present in gauge field theories (in positive gauges). This has been explicitly shown for the Schwinger model, usually regarded as a prototype of gauge theories with unbroken gauge symmetry, [6] and the present paper provides a detailed analysis of the Kibble model, in four dimensions, usually regarded as the prototype of gauge theories exhibiting the Higgs phenomenon [7–9]. The model can be obtained from the abelian Higgs-Kibble model by neglecting the quantum fluctuations of the modulus of the Higgs field $\chi=|\chi|e^{i\varphi}$ and by actually freezing $|\chi|$ to 1. In this case the Higgs-Kibble Lagrangian becomes

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}^2-(\partial_\mu\varphi-eA_\mu)^2+\text{gauge fixing} \tag{1.1}$$

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