Commun. Math. Phys. 111, 555–577 (1987)

Communications in Mathematical Physics

One-Dimensional Random Ising Systems with Interaction Decay $r^{-(1+\varepsilon)}$: A Convergent Cluster Expansion

M. Campanino[1] and E. Olivieri[2,*]

[1] Dipartimento di Matematica, II Università di Roma "Tor Vergata", Roma, Italy – G.N.A.F.A. – C.N.R.

[2] Scuola Internazionale Superiore di Studi Avanzati, Trieste, Italy

Abstract. We consider a one-dimensional random Ising model with Hamiltonian

$$H=\sum_{i\neq j}\frac{J_{ij}}{|i-j|^{1+\varepsilon}}s_i s_j+h\sum_i s_i\ ,$$

where $\varepsilon>0$ and J_{ij} are independent, identically distributed random variables with distribution $dF(x)$ such that

i) $\int x\,dF(x)=0$,

ii) $\int e^{tx}dF(x)<\infty \quad \forall t\in\mathbb{R}$.

We construct a cluster expansion for the free energy and the Gibbs expectations of local observables. This expansion is convergent almost surely at every temperature. In this way we obtain that the free energy and the Gibbs expectations of local observables are C^∞ functions of the temperature and of the magnetic field h. Moreover we can estimate the decay of truncated correlation functions. In particular for every $\varepsilon'>0$ there exists a random variable $c(\omega)$, finite almost everywhere, such that

$$|\langle s_0 s_j\rangle_H-\langle s_0\rangle_H\langle s_j\rangle_H|\leqq\frac{c(\omega)}{|j|^{1+\varepsilon-\varepsilon'}}\ ,$$

where $\langle\ \rangle_H$ denotes the Gibbs average with respect to the Hamiltonian H.

1. Introduction, Definitions and Results

In [4] a one-dimensional Ising spin system with random interactions decaying like $1/r^{1+\varepsilon}$ was considered. A weak version of uniqueness of Gibbs state was proven there for such a system by showing that at every temperature the expectation of an

* Permanent address: Dipartimento di Matematica, Università di Roma "La Sapienza", Piazzale A. Moro 2, Roma, Italy – G.N.F.M. – C.N.R.