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Modular Invariance for Interacting Bosonic Strings at Finite Temperature

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Abstract. We study the finite temperature string path integral introduced by Polchinski [1]. It is shown that on an arbitrary genus world sheet all windings of the fields around the compact time direction can be rotated into a single cycle. The modular invariance of this result is demonstrated.

I. Introduction

The recent work [2] on strings at finite temperature has so far considered free strings in flat spacetime. Since the extended structure of strings will only become apparent near the Planck scale, the most likely setting for obtaining (indirect) evidence is the observation of cosmological effects from the early universe. A most remarkable property is the existence of the Hagedorn limiting temperature [3] at which the free energy diverges. Although this critical temperature has been shown only in the ideal gas approximation, it signals the likelihood that string cosmology differs drastically from the standard big bang scenario in the early stages.

During the Planck epoch it is impossible to ignore the extreme conditions which exist; it is expected that any perturbative expansion will then be invalid. Nevertheless, it is still interesting to consider the interactions of strings in a regime where perturbation theory holds. It is therefore the aim of this paper to show how to include interactions for the closed bosonic string in 26 dimensions. For simplicity, curvature effects will not be considered. In this regard, our result should be considered more as a lowest order contribution for more general backgrounds when an adiabatic approximation is used. The starting point will be the Polyakov path integral as discussed by Polchinski [1].

In [1], the free energy density $F(\beta)$ is calculated for a gas of a variable number of non-interacting closed strings. In other approaches [2], which focus on calculating the Hagedorn temperature, the partition function for a single string (in a heat bath) is derived as for a system of independent oscillators. With the inclusion of interactions, though, a more systematic treatment is needed.

The requirement of modular invariance of physical quantities—here, the free energy—shall play a key role in this formulation. In particular, the embeddings of a