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Classical Lattice Gas Model with a Unique Nondegenerate but Unstable Periodic Ground State Configuration

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Abstract. We construct a classical lattice gas model with a unique periodic ground state configuration such that the Peierls' condition is not satisfied. The ground state configuration is nondegenerate, which means that for any fixed energy E and any integer n, the diameter of the support of all n-connected local excitations, with energy less than E, is bounded. Nevertheless the configuration is not stable: it does not give rise to a low temperature phase. Any translation invariant Gibbs state of our model corresponds to quasiperiodic ground state configurations. This requires the modification of a recent hypothesis of Dobrushin and Shlosman.

1. Introduction

In their recent paper Dobrushin and Shlosman [1] formulated a hypothesis concerning the stability of ground state configurations of classical lattice systems. They addressed the question: which ground state configurations give rise to the low temperature phases? One of their suggestions was the following. Assume all periodic ground state configurations be related by the symmetry of the Hamiltonian. Then a ground state configuration should be stable if and only if it is nondegenerate, where by nondegeneracy it is meant that for any fixed energy E and any integer n the diameter of the support of all n-connected local excitations with energy less than E is bounded.

Here we provide a counterexample to this assertion. Our model is a two-dimensional lattice gas system with nearest neighbor interaction. The construction is based on Robinson's tiles [2, 3]. There is a family of 56 square-like tiles which tile the plane only in a nonperiodic fashion. This can be translated into a lattice gas model without any periodic ground state configurations in the following way [4, 5, 3]. Every site of the square lattice can be occupied by one of the 56 different particle-tiles. Two nearest neighbor particles which do not "match" contribute positive energy; otherwise the energy is zero. Such model does not have periodic ground state configurations. It does possess, however, "quasiperiodic" ground state configurations. A configuration of particles is quasiperiodic if when a certain