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A Topological Investigation of the Quantum Adiabatic Phase*

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Abstract. Using algebraic topology, the appearance of the Quantum Adiabatic Phase over various parameter manifolds is investigated. The relation with non-trivial gauge bundles (both abelian and non-abelian) is studied and it is shown that the phase appears as a result of homotopically non-trivial mappings, induced by the Hamiltonian in the space of wave-functions. The cohomological picture is developed and some topological considerations concerning field theory anomalies in the Hamiltonian picture are presented. A proof of the Nielsen-Ninomiya theorem is given inspired from the notion of the adiabatic phase.

1. Introduction

Recently, M. Berry found that the usual form of the Quantum Adiabatic Theorem was not exactly correct, [1]. He showed that when the Hamiltonian depends on several parameters that change adiabatically with time and the topology of the manifold spanned by the parameters is non-trivial (usually induced by accidental degeneracies), there is another contribution to the phase acquired by the wave-function of the system (under adiabatic transport). This is topological in nature and it is related to the first Chern class of the natural hermitian (abelian) connection in the Hilbert bundle over the parameter manifold [2].

Later, Wilczek and Zee, [3], generalized the notion of the adiabatic phase to the non-abelian case corresponding to adiabatically transporting an n-fold degenerate state over the entire parameter manifold.

This discovery turned out to be very fruitful in a lot of situations, including applications to diatoms [4], modifications of the Bohr-Sommerfeld semiclassical quantization in some special cases [5], the explanation of the quantum Hall effect

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