Commun. Math. Phys. 111, 393–407 (1987)

Communications in Mathematical Physics
© Springer-Verlag 1987

The Boltzmann Equation for Weakly Inhomogeneous Data

L. Arkeryd[1], R. Esposito[2]* and M. Pulvirenti[2]*

1 Department of Mathematics Chalmers University of Technology and the University of Goteborg, Sweden
2 Dipartimento di Matematica Universita, "La Sapienza" di Roma, Italy.

Abstract. We solve the initial value problem associated to the nonlinear Boltzmann equation in the case in which the initial distribution has sufficiently small spatial gradients.

1. Introduction and Notation

The nonlinear Boltzmann equation is believed to describe the time evolution of a rarefied gas of particles. It takes the form

$$D_t f := (\partial_t + v \cdot \nabla_x) f = Q(f,f), \quad f(0) = f_0, \tag{1.1}$$

where $D \times \mathbb{R}^3 \ni (x, v) \to f(x, v)$ is the distribution of the particles, x and v denote position and velocity respectively, $D \subset \mathbb{R}^3$ is the domain to which the gas is confined. The bilinear operator Q takes into account the interaction among the particles and will be specified later.

The initial value problem (1.1) has been considered by several authors, and the following three groups of results global in time are available;

a) *Spatially Homogeneous case.* If f_0 depends only on v, f_t has still this property. In this case global existence and uniqueness result can be proved for very general data.

b) *Small Deviations from Equilibrium.* It is well known that the nontrivial equilibria for the problem (1.1) are the Maxwellian distributions. An initial distribution f_0, slightly differing from a Maxwellian, can be proved to evolve globally and uniquely in time, according to Eq. (1.1). Moreover it approaches a Maxwellian asymptotically in time.

c) *Small Deviations from Vacuum.* A global existence and uniqueness theorem for the initial value problem (1.1) can be proved for an initial distribution $f_0(x, v)$, $x \in \mathbb{R}^3$,

* Supported by C.N.R.–GNFM and MPI