Commun. Math. Phys. 111, 373–392 (1987)

Communications in Mathematical Physics

A Superanalog of the Selberg Trace Formula and Multiloop Contributions for Fermionic Strings

A. M. Baranov[1], Yu, I. Manin[2], I. V. Frolov[3], and A. S. Schwarz[1]

[1] Institute of Physical Engineering, Moscow
[2] Steklov Mathematical Institute, Moscow
[3] Institute of Minerals and Raw Materials, Moscow

Abstract. An analog of the classical Selberg trace formula is given for discrete groups, acting on the upper complex half-superplane. Applications to the fermionic string measure on the moduli superspace are discussed.

Introduction

Quantum string theory is being discussed now with growing interest both by physicists and mathematicians. Common expectations are that one of the superstring models will correctly describe Planck scale physics and, after multiple symmetry breaking, will lead to the well known effective lagrangians in the low-energy limit.

Mathematically, quantum string theory exists in two versions. Much work was done with canonical quantization, which opened a very interesting new chapter of representation theory. However, it is notably difficult to construct a consistent picture of interacting strings in the operator approach. The Polyakov path integral formalism [1] is devoid of this shortcoming, since to account for interactions in this approach it suffices to sum over all world sheets with topologies compatible with boundary conditions. In particular, the partition function and the amplitudes in the critical dimension $d=26$ are expressed as a sum of a series, where the g-loop contribution is given by an integral over the moduli space M_g of conformal classes of Riemannian surfaces of genus g. In this way a measure on M_g arises, which is expressed as a certain combination of determinants of the Laplace operators (cf. [2]). For $g>1$ the conformal moduli space M_g may be considered as a space of surfaces of constant negative curvature. By means of this identification, one can express the Polyakov measure on the moduli space through the geometric invariants of the constant curvature metrics, namely lengths of geodesics (cf. [3]). To this end one uses the Selberg trace formula [4] and certain recent results on the Weil-Petersson metric of the moduli space. These formulas, announced in [3], are briefly reviewed in Sect. 2 of this paper. A detailed presentation with slightly more precise formulas has since appeared in [5]. An objective of this work is to develop similar tools for the quantum fermionic string theory in the critical dimension