Multicomponent Composites, Electrical Networks and New Types of Continued Fraction II

G. W. Milton[*]

California Institute of Technology, 405-47, Pasadena, CA 91125, USA

Abstract. The outstanding problem of systematically developing rigorous bounds on the complex effective conductivity tensor σ^* of d-dimensional, n-component composites with $n > 2$ is solved. The bounds incorporate information contained in successively higher order correlation functions which reflect the composite geometry. Explicit expressions are given for many of the bounds and some, but not all of them, are represented by nested sequences of circles in the complex plane that enclose, and in fact converge to, each diagonal element of σ^*. They are derived from the fractional linear matrix transformations found in Part I that recursively link σ^* with a hierarchy of complex effective tensors $\Omega^{(j)}$, $j = 0, 1, 2, \ldots$, of increasing dimension, $d(n-1)^j$. Elementary bounds on $\Omega^{(j)}$ confining the diagonal elements of $\Omega^{(j)}$ or its inverse to half-plane, wedge or open polygon regions of the complex plane, imply narrow bounds on σ^* which converge to the exact value of σ^* in the limit as $j \to \infty$. When the component conductivities are real these bounds are more restrictive than the corresponding variational bounds. Besides applying to the effective conductivity σ^*, the bounds extend to a wide class of matrix-valued multivariate functions called Ω-functions, and thereby to conduction in polycrystalline media, viscoelasticity in composites, and conduction in multi-component, multiterminal, linear electrical networks. The analytic and invariance properties of Ω-functions are explored and within this class of function most of the bounds are found to be optimal or at least attainable. The bounds obtained here are essentially a generalization to matrix-valued, multivariate functions of the nested sequence of lens-shaped bounds in the complex plane derived by Gragg and Baker for single variable Stieltjes functions.

[*] Address after January 15, 1987: Courant Institute, New York University, 251 Mercer St., New York, NY 10012, USA