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Multicomponent Composites, Electrical Networks and New Types of Continued Fraction I

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Abstract. The development of bounds on the complex effective conductivity tensor σ^* (that relates the average current to the average electric field in a multicomponent composite) has been hindered by lack of a suitable continued-fraction representation for σ^*. Here a new field equation recursion method is developed which gives an expression for σ^* as a continued fraction of a novel form incorporating as coefficients the component conductivities and a set of fundamental geometric parameters reflecting the composite geometry. A hierarchy of field equations is set up such that the solutions of the $(j+1)$th-order equation generate the solutions of the jth-order equation. Consequently the effective tensor $\Omega^{(j)}$ associated with the jth-order field equation is expressible as a fractional linear matrix transformation of $\Omega^{(j+1)}$. These transformations combine to form the continued fraction expansion for $\sigma^* = \Omega^{(0)}$ which is exploited in the following paper, Part II, to obtain bounds: crude bounds on $\Omega^{(j)}$, for $j \geqq 1$, give narrow bounds on σ^*. The continued fraction is a generalization to multivariate functions of the continued fraction expansion of single variable Stieltjes functions that proved important in the development of the theory of Páde approximants, asymptotic analysis, and the theory of orthogonal polynomials in the last century. The results extend to other transport problems, including conduction in polycrystalline media, the viscoelasticity of composites, and the response of multicomponent, multiterminal linear electrical networks.

1. Introduction

A central problem in Physics is the evaluation of the macroscopic response of a system given the formulae governing its microscopic behavior. Here a new *field equation recursion method* is introduced for estimating the effective transport

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