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Point Singularities of Coupled Gauge Fields with Low Energy

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Abstract. Isolated singularities in higher-dimensional Yang-Mills-Higgs fields are considered. The singularities are removable if the energy is smaller than a dimensional constant.

Introduction

Conditions under which Yang-Mills-Higgs fields cannot have isolated singularities are now know for arbitrary dimension [5–11]. In all cases the Yang-Mills field F is required to be an element of the space $L^{n/2}$, where n is dimension; the Higgs field ϕ must satisfy certain L^p conditions depending on n and on a physical parameter λ. (In dimension 2 an additional holonomy condition is necessary [9].) The arguments depend crucially on the conformal invariance of the norm $\|h\|_{n/2}$, where

$$h^2 = |F|^2 + |\nabla\phi|^2 + |\phi|^4$$

is the density of the energy integral

$$E = \int h^2 .$$

An example is known of a pure Yang-Mills field ($\phi = 0$) in which $h \in L^p$ for any $p < n/2$ and h is singular. In this example, which is defined by pulling back a bundle-valued F on S^{n-1}, $h(x) \sim \text{const}/|x|^2$. Thus the energy density tends to infinity near $x = 0$ and its L^2 norm, though finite, is not necessarily small. (The example is due to Uhlenbeck and is reported in [7].)

It is reasonable to ask whether the physically natural condition of low energy is sufficient for removing point singularities. One would then hope to show directly that the asymptotically high energy of Uhlenbeck's example accounts for its nonremovable singularity. It would be sufficient to show that h is in the space $L^{n/2}$

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