Commun. Math. Phys. 111, 151–160 (1987)

Communications in Mathematical Physics

The Growth Rate for the Number of Singular and Periodic Orbits for a Polygonal Billiard

A. Katok*

Department of Mathematics, California Institute of Technology, Pasadena, CA 91125, USA

Abstract. For any simply connected polygon in the plane, the number of billiard orbits which begin and end at a vertex grows subexponentially with respect to the length or to the number of reflections. This implies that the numbers of isolated periodic orbits and of families of parallel periodic orbits do grow subexponentially. The main technical device is a calculation showing that the topological entropy of the Poincaré map for the billiard flow is equal to zero.

Let $\Delta \subset \mathbb{R}^2$ be a simply connected polygon. A broken (polygonal) line formed by the segments $[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]$ will be called a *generalized diagonal* of Δ if it lies inside Δ except for the points $x_0, \ldots, x_n$, the points x_0 and x_n are vertices of Δ, the points $x_1 \ldots x_{n-1}$ lie on the sides of Δ, and for $i=1, \ldots, n-1$ the segments $[x_{i-1}, x_i]$ and $[x_i, x_{i+1}]$ form the same angle with the side of Δ passing through x_i (cf. Fig. 1).

The total number of different generalized diagonals of Δ is always infinite. Let $D_T(\Delta)$ be the number of different generalized diagonals of Δ of length $\leqq T$.

The purpose of the first three sections of this note is to prove the following elementary geometric theorem:

Theorem. $\lim\limits_{T \to \infty} \dfrac{\log(D_T(\Delta))}{T} = 0.$

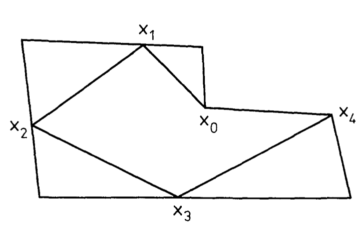

Fig. 1

* Supported in part by NSF Grant #DMS-8414400