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# Asymptotic Completeness and Multiparticle Structure in Field Theories

## II. Theories with Renormalization: The Gross-Neveu Model

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**Abstract.** The ideas developed in Part I (ref. [1]) are applied to the recently constructed massive Gross-Neveu model. We define in this case an irreducible kernel satisfying a regularized Bethe-Salpeter equation which is convenient to derive asymptotic completeness in the 2-particle region. As in Part I, the method allows direct graphical definition of general irreducible kernels and is well suited to the analysis of asymptotic completeness and related results in more general energy regions.

A large part of the paper is devoted to a new self-contained construction (via phase space expansion) of the Gross-Neveu model. The presentation is somewhat simpler than previous ones, is more complete on some points and is best suited to our purposes.

## 1. Introduction

In all models that involve renormalization, a phase-space analysis [2–7] is needed, e.g. to control ultraviolet divergences. A method that has proved convenient is to introduce a suitable decomposition of momentum space into slices with, for each slice, cluster expansions with a conveniently scaled lattice. An analogue of the cluster expansion, with respect to momentum slices, is also a priori needed. However, a somewhat simpler method can be applied in fermionic theories such as the massive Gross Neveu model [0], which is a fermionic model in 2 dimensions with quartic interaction a colour number $\geqq 2$, and which is asymptotically free. In fact, in contrast to bosonic models, it is useful to expand the exponential of the interaction which is of the form

$$\exp\left[\lambda \int_{\Lambda} (\bar{\psi}\psi)^2(y)dy\right] \quad \text{as a sum} \quad \sum \frac{\lambda^n}{n!}\left[\int_{\Lambda} (\bar{\psi}\psi)^2(y)dy\right]^n .$$

Each field is decomposed into fields $\psi^{(i)}$ depending on the momentum slice $i$, and cluster expansions are now applied for each $i$ to

$$\text{integrals of the form } \int \prod_v \psi^{(i)}(y_v) \prod_w \bar{\psi}^{(i)}(y_w) d\mu(\bar{\psi}^{(i)}\psi^{(i)}) .$$