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On the Quotient of the Regularized Determinant of Two Elliptic Operators*

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Abstract. We study the quotient of the regularized determinants of two elliptic operators having the same principal symbol. We prove that, under general conditions, a method recently proposed by Tamura coincides with the ζ-function approach.

1. Introduction

In the computation of quadratic path-integrals one is led to the evaluation of determinants of elliptic operators. Nevertheless, these determinants diverge. Hence, it is necessary to adopt some regularization procedure.

Often, it is the quotient of the determinants of two operators D_0 and D_1 that is searched. In this case, one can attempt to connect them by means of a differentiable one-parameter family D_t of operators, and if the determinant regularized according to some prescription results in a differentiable function of the parameter t, the quotient can be computed as the exponential of an integral, i.e.

$$\frac{\det(D_1)}{\det(D_0)} = \exp \int_0^1 \frac{d}{dt} \log(\det(D_t))\, dt\,. \tag{1.1}$$

One regularizing prescription that can be used in this approach is the well-known ζ-function method [1], since it has the required differentiability [2, 3].

Recently, Tamura [4] proposed an alternative method to regularize the determinant of the radio of two Dirac operators D_0 and D_1 based on Fujikawa's results [5]. In this approach, it is not the value of the determinant of each operator that is given, indeed it is the change of the logarithm of the determinant that is regularized. In order to do it, he defines the M depending function

$$D_M(D_1; D_0) = \exp\left[\operatorname{Tr} \int_0^1 \frac{dD_t}{dt} D_t^{-1} \exp(-D_t^2/M^2)\, dt\right], \tag{1.2}$$

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