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Wave Operators and the Incompressible Limit of the Compressible Euler Equation

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Abstract. In an exterior domain in R^n $(n \geqq 2)$, the solution of the compressible Euler equation is shown to converge to that of the incompressible Euler equation when the Mach number tends to 0. The initial layer appears.

1. Introduction

In our previous work [8], we have shown that the solution of the compressible Euler equation in an exterior domain in R^3 converges to that of the incompressible Euler equation when the Mach number tends to 0 even if the initial velocity is not divergence free. The aim of this article is to generalize this result for R^n $(n \geqq 2)$ and also to provide a simpler proof.

We consider the movement of an ideal fluid in a domain Ω in R^n $(n \geqq 2)$ exterior to a bounded obstacle. Let P be its pressure and V the velocity. Then the Euler equation is written as

$$\begin{aligned}
&\partial_t P + (V \cdot \nabla)P + \gamma P \nabla \cdot V = 0, \\
&\partial_t V + (V \cdot \nabla)V + \lambda^2 P^{-1/\gamma} \nabla P = 0, \\
&\nu \cdot V = 0 \quad \text{on} \quad S,
\end{aligned}$$

where $\partial_t = \partial/\partial t$, γ is a constant >1, S is the boundary of Ω and ν is the outer unit normal to S. λ is a large parameter proportional to the inverse of the Mach number (see, e.g., [14, p. 52]). We assume that S is smooth and Ω is arcwise connected, but nothing is assumed on the shape of the boundary. It is convenient to transform the dependent variable P into $Q = \dfrac{\gamma}{\gamma - 1} P^{1-1/\gamma}$. Then the above equation can be rewritten as

$$\begin{aligned}
&\partial_t Q + (V \cdot \nabla)Q + (\gamma - 1)Q \nabla \cdot V = 0, \\
&\partial_t V + (V \cdot \nabla)V + \lambda^2 \nabla Q = 0.
\end{aligned}$$

We set $\gamma = 2$ for the sake of simplicity. We want to assume that the initial pressure has an asymptotic expansion of the form: Const. $+ O(\lambda^{-1})$. Therefore, without loss