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Convergence to Diffusion Waves of Solutions for Viscous Conservation Laws

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Abstract. We study the large-time behavior of solutions of viscous conservation laws. It is shown that solutions tend to diffusion waves, which are constructed based on the heat equation and Burgers equation. The convergence is in the L_p, $1 \leqq p \leqq \infty$ sense and is obtained as a consequence of the L_2 decay of the difference between the solution and its asymptotic state of diffusion waves.

1. Introduction

Consider a system of viscous conservation laws

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = \frac{\partial}{\partial x}\left(B(u)\frac{\partial u}{\partial x}\right), \qquad u \in \mathbb{R}^n, \qquad t \geqq 0, \qquad -\infty x < \infty . \tag{1.1}$$

We are interested in the large-time behavior of solutions whose initial value tends to a constant state at $x = \pm\infty$. Without loss of generality we take the constant state to be zero:

$$u(x,0) \to 0 \quad \text{as} \quad x \to \pm\infty . \tag{1.2}$$

Physical models of the form (1.1) include the compressible Navier-Stokes equations and magnetohydrodynamics. The $n \times n$ *viscosity* matrix $B(u)$ represents a dissipative mechanism, and the solution $u(x,t)$ is expected to decay to the zero state as $t \to \infty$. The decay in L_∞ and L_2 has been studied by viewing (1.1) as a perturbation of the linearized equations

$$\frac{\partial u}{\partial t} + f'(0)\frac{\partial u}{\partial x} = B(0)\frac{\partial^2 u}{\partial x^2}, \tag{1.3}$$

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