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Convergence of the $U(1)_4$ Lattice Gauge Theory to Its Continuum Limit

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Abstract. It is shown that in four space-time dimensions the compact $U(1)$ lattice gauge theory with general energy function converges to a renormalized free electromagnetic field on the current sector as the lattice spacing approaches zero, provided the coupling constant is sufficiently large. For the Wilson energy function, it is possible, by judicious choice of the Gibbs state, to get convergence for arbitrary coupling strengths. Furthermore, for all but a countable number of values of the coupling constant, the limit exists and is independent of the particular state chosen to define the lattice model.

1. Introduction

The problem of mathematical existence of quantized Yang–Mills' fields is (on an informal level) equivalent to defining a certain probability measure on a space of connection forms. The informal description of this (Yang–Mills') measure is

$$d\mu(A) = Z^{-1}\exp\left[\frac{1}{2g^2}\int_{\mathbb{R}^d}\sum_{i<j}\operatorname{trace}(F_{ij}^A(x)^2)dx\right]DA, \tag{1.1}$$

where A runs over a space of connection forms $(\underline{A})$ on the trivial unitary vector bundle $\mathbb{C}^N \times \mathbb{R}^d$, $F^A = dA + A \wedge A$ is the curvature of A, $DA = \prod_{i=1}^{d}\prod_{x\in\mathbb{R}^d} d(A_1(x))$ is "infinite dimensional Lebesgue measure" on $(\underline{A})$, g^2 is a positive "coupling" constant, and Z is a normalization constant which makes μ a probability measure. See Gross [5] for a discussion of (1.1) and its ailments.

A standard approach for trying to make sense of the informal expression (1.1) is to "approximate" the measure by a compact lattice gauge model introduced by K. Wilson [1], see Sect. 3 below. The problem is then to show that the lattice measures have a limit as the lattice spacing tends to zero (the continuum limit).

The procedure for removing the lattice cutoff has still not been carried out for space-time dimension larger than two with a non-abelian gauge (structure) group.

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