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The Spectrum of Operators Elliptic Along the Orbits of $\mathbb{R}^n$ Actions

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Abstract. It is shown that a periodic elliptic operator on $\mathbb{R}^n$ has no eigenvalues off of the set of discontinuities of its spectral density function. The methods involve operator algebras and are based on a "spectral duality" principal first introduced by J. Bellisard and D. Testard. A version of the spectral duality theorem is proved which relates the point spectrum of a certain family of operators to the continuous spectrum of an associated family.

0. Introduction

A general method for studying elliptic operators on open manifolds involves viewing such an operator as a longitudinally elliptic operator on a foliated compact manifold having the original open manifold as a leaf. This is the point of view developed by A. Connes [4]. We shall use this approach to study properties of the spectrum of such an operator. Our work is an extension of the ideas and methods introduced by Bellisard and Testard in [2]. The principal tool is a variant of their "spectral duality theorem." We prove a version of this theorem which holds for operators on the orbits of a locally free action of $\mathbb{R}^n$ on a compact Hausdorff space. The main application states that a periodic elliptic operator on $\mathbb{R}^n$ has no eigenvalues, off of the discontinuities of the spectral density function, when considered as acting on $L^2(\mathbb{R}^n)$.

The paper is organized as follows. Section 1 introduces the necessary operator algebra and discusses two families of representations of it. In Sect. 2 the basic facts on operators elliptic along the orbits of the $\mathbb{R}^n$ action are presented. The spectral duality theorem is proved in Sect. 3 and the application discussed above to the spectrum of periodic elliptic operators on $\mathbb{R}^n$ is covered in Sect. 4.

We remark that it will be quite useful to have a version of the spectral duality theorem which will hold for more general foliations. The case of a foliation transverse to the fibers of a principal G-bundle is particularly appropriate, and we will consider this in a later publication.

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