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Convergence of Local Charges and Continuity Properties of W^*-Inclusions*

C. D'Antoni[1], S. Doplicher[1], K. Fredenhagen[2], and R. Longo[1]

[1] Universitá di Roma, La Sapienza, Dipartimento di Matematica, I-00185 Roma, Italy
[2] II. Institut für Theoretische Physik, Universität Hamburg, D-2000 Hamburg 50, Federal Republic of Germany, Heisenberg fellow

Abstract. The local generators of symmetry transformations which have recently been constructed from a quantum field theoretical version of Noether's theorem are shown to converge to the global ones as the volume tends to the whole space. The proof relies on the continuous volume dependence of the universal localizing maps which are associated to the local split W^*-inclusions.

1. Introduction

A new approach to a quantum Noether theorem has recently been set up [1–3] in the algebraic formulation of quantum field theory [4]. In a given theory, let $\mathscr{F}(\mathcal{O})$ be the von Neumann algebra which is generated by the field operators which are localized in the bounded space-time region $\mathcal{O}$. Let G be the group of space-time and internal symmetries, and let J_u for each u in the Lie algebra $\mathscr{G}$ of G denote the corresponding selfadjoint generator of the global symmetry transformation. The quantum Noether theorem then asserts that there exist local field operators which induce that symmetry locally.

The construction of these local generators is based on the so-called split property [5] (see below) which may be understood as the possibility to decouple a region $\mathcal{O}$ completely from any other region which is separated from $\mathcal{O}$ by a finite spacelike distance. Assume that $\mathscr{F}$ possesses the split property, and let $\mathcal{O}$ and $\hat{\mathcal{O}}$ be bounded space-time regions such that $\mathcal{O}+x\subset\hat{\mathcal{O}}$ for all x in some neighbourhood of the origin. Then for each $u\in\mathscr{G}$ there is a selfadjoint operator $J_u^{\mathcal{O},\hat{\mathcal{O}}}$ which is affiliated to $\mathscr{F}(\hat{\mathcal{O}})$ and induces on $\mathscr{F}(\mathcal{O})$ the infinitesimal symmetry transformation u, i.e.

$$e^{i\lambda J_u^{\mathcal{O},\hat{\mathcal{O}}}}\in\mathscr{F}(\hat{\mathcal{O}}),\qquad \lambda\in\mathbb{R}, \tag{1.1}$$

and for sufficiently small λ

$$e^{i\lambda J_u^{\mathcal{O},\hat{\mathcal{O}}}}Fe^{-i\lambda J_u^{\mathcal{O},\hat{\mathcal{O}}}}=e^{i\lambda J_u}Fe^{-i\lambda J_u}. \tag{1.2}$$

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