

A Phase Cell Approach to Yang-Mills Theory

III. Local Stability, Modified Renormalization Group Transformation

Paul Federbush

University of Michigan, Department of Mathematics, Ann Arbor, MI 48109, USA

Abstract. In this paper the basic local stability result is obtained, in a form valid in both small field and large field regions. To achieve this, some modifications are made in both the action and the renormalization group transformation. Though there is some sacrifice of elegance in these modifications, the establishment of this local stability estimate yields the most basic ingredient of the phase cell cluster expansion, good estimates for all the actions.

Incidental to the estimates of this paper we establish some results on “lattice geometry,” interesting in their own right. A bound on the “minimum area” of a loop of length l , in d dimensions, is obtained as $\frac{l^2}{8} \left(1 - \frac{1}{d}\right)$. This, a best possible bound, was obtained for us by A. Blass. We also construct a “radial” maximal tree for the lattice in d dimensions. We hope to stimulate someone to find a better construction of “radial” trees.

Introduction

It is not far amiss to say that each machine in Constructive Quantum Field Theory has two essential ingredients, a perturbative aspect (to handle renormalization cancellations) and a positivity or stability aspect (the non-perturbative feature). This latter occurs under different names in different programs: positivity of the vacuum energy in the traditional cluster expansion; the bounds on partition functions in the method of exact renormalization transformations; and α -positivity and α -stability in the phase cell cluster expansion approach to boson models. In this paper we establish essential stability results for our phase cell attack on Yang-Mills theories. (This may have been the most difficult problem we have to face.) The ideas in this paper may also be useful in other approaches to the study of four dimensional gauge theories.