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The Generalized Plasma in One Dimension: Evaluation of a Partition Function

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Abstract. Forrester and Jancovici have given sum rules for a two-dimensional generalized plasma with two species of particles interacting through logarithmic potentials with three independent coupling constants. They have also found a specific one-dimensional solvable model which satisfies the analogs of their sum rules. A class of one-dimensional models for which the partition function is evaluable is given as well as a more general result evaluating multi-dimensional integrals.

1. Introduction

Forrester and Jancovici [1] have given an exactly solved model for a one-dimensional generalized plasma with two species of particles (roman and greek), interacting through logarithmic potentials, with three independent coupling constants. This was motivated by their discovery of sum rules for such a generalized plasma in two dimensions (see also Halperin [2] and Girvin [3]) and the desire to at least verify the one-dimensional analogs of these rules.

The two-dimensional system with roman and greek particles of density ϱ_R and ϱ_G, respectively, and independent coupling constants g_{RR}, g_{RG}, g_{GG} has Hamiltonian

$$\begin{aligned} H = & -g_{RR}\sum_{i>j}\ln r_{ij} - g_{GG}\sum_{\alpha>\beta}\ln r_{\alpha\beta} - g_{RG}\sum_{i,\alpha}\ln r_{i\alpha} \\ & + (g_{RR}\varrho_R + g_{RG}\varrho_G)\sum_i \int \ln|\mathbb{R}_i - \mathbb{R}|\,d\mathbb{R} \\ & + (g_{GG}\varrho_G + g_{RG}\varrho_R)\sum_\alpha \int \ln|\mathbb{R}_\alpha - \mathbb{R}|\,dr \\ & - (\tfrac{1}{2}g_{RR}\varrho_R^2 + \tfrac{1}{2}g_{GG}\varrho_G^2 + g_{RG}\varrho_R\varrho_G)\int \ln|\mathbb{R} - \mathbb{R}'|\,d\mathbb{R}\,d\mathbb{R}', \end{aligned} \tag{1.1}$$

where the particle-background and background-background interactions have been chosen in a way which compensates the remote particle-particle interactions

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