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Continuum Limit of a Hierarchical SU(2) Lattice Gauge Theory in 4 Dimensions

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Abstract. We study nonperturbative renormalizability of a $d=4$ hierarchical SU(2) gauge model that realizes Migdal's recursion relation as an exact renormalization group transformation. A continuum limit of effective actions is shown to exist as the scaling limit, both for initial Wilson and heat kernel actions. These limit effective actions exhibit ultraviolet asymptotic freedom and provide a strictly positive string tension.

1. Introduction

The study of nonperturbative renormalization of gauge theories is presently one of the major efforts within constructive quantum field theory. To employ for this aim the framework of lattice quantization [1] looks very appealing – in spite of its obvious shortcomings – since it preserves the characterizing property of such theories: local gauge invariance. Moreover Wilson's [2] renormalization group approach to remove the ultraviolet cutoff introduced by the lattice, turned out to be an extraordinarily attractive suggestion. From a physical point of view the 4-dimensional Yang-Mills theory is of central interest. Although there is important progress in the rigorous construction of this theory [3, 4] a complete solution has not yet been achieved. The study of the ultraviolet behaviour of this asymptotically free theory by exact renormalization group transformations allows us to make use of a perturbation expansion but requires i) a control of higher order contributions, ii) a nonperturbative treatment of the "large fields," and iii) a control of the nonlocal effective actions generated. Shortly after the pioneering work of Wilson [1] on lattice gauge theories, Migdal [5] proposed a recursion relation considered as an approximate real space renormalization group transformation both for spin systems and lattice gauge theories. In the case of spin systems it was realized some time ago that this recursion relation holds exactly on hierarchical lattices [6], yet only recently Ito [7] presented hierarchical lattice gauge models with similar properties. In view of the enormous complexity to be mastered attacking the full $d=4$ nonabelian gauge theory, we pursue the modest aim to