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Maximal Violation of Bell's Inequalities is Generic in Quantum Field Theory

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Abstract. Under weak technical assumptions on a net of local von Neumann algebras $\{\mathscr{A}(\mathscr{O})\}$ in a Hilbert space $\mathscr{H}$, which are fulfilled by any net associated to a quantum field satisfying the standard axioms, it is shown that for every vector state ϕ in $\mathscr{H}$ there exist observables localized in complementary wedge-shaped regions in Minkowski space-time that maximally violate Bell's inequalities in the state ϕ. If, in addition, the algebras corresponding to wedge-shaped regions are injective (which is known to be true in many examples), then the maximal violation occurs in any state ϕ on $\mathscr{B}(\mathscr{H})$ given by a density matrix.

I. Introduction

With the discovery of Bell's inequalities [6, 7] and their subsequent refinements [10, 24, 26, 32], it became possible for the first time to test experimentally whether or not certain concepts of classical physics can be used in a description (at least within a fairly general metatheoretic framework) of processes that take place on an atomic or subatomic scale. The inequalities concern statistical correlations of measurements made on two parts of one system and are derived from two basic assumptions. One is that all measured correlations $p(\alpha, \beta)$ between outcomes α measured on one subsystem and outcomes β measured on the other subsystem can be modelled within a classical probabilistic theory (which subsumes, of course, deterministic theories). The second is that this description is "local" in the sense that the choice of a measuring device operating upon one part of the system does not affect the probabilities of outcomes measured on the other part. For a more thorough discussion of the assumptions and the metatheoretical framework within which Bell's inequalities can be derived, see [32].

It is not our intent to enter into a philosophical discussion of Bell's inequalities in this forum. Our aim is to show that, just as quantum mechanics does, quantum field theory (QFT) predicts a maximal violation of Bell's inequalities, and that in fact, unlike quantum mechanics, QFT predicts that this maximal violation is generic in a sense we shall explain.