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Mass Generations in Two-Dimensional Hierarchical Heisenberg Model of Migdal-Kadanoff Type

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Abstract. Finiteness of correlation length in the 2D Heisenberg model is established within the Migdal-Kadanoff approximate renormalization recursion formulas.

1. Introduction

Though it is well known that the two-dimensional (2D) $O(N)$-invariant statistical mechanical models ($N \geqq 2$) with short range interactions do not exhibit any long-range orders [4, 18], we still do not have a rigorous proof for our longstanding conjecture that the 2D $O(N)$-invariant statistical mechanical models would always have non-zero masses for all inverse temperatures $\beta \geqq 0$, provided N is larger than or equal to 3. This problem is rather similar to another longstanding problem of quark confinements in 4D non-Abelian lattice gauge theories [19, 1, 5]. These problems are probably solved only by hard analysis like real space renormalization group methods [6, 14, 20]. Unfortunately, the real space renormalization group methods are sometimes very complicated. Thus some approximate renormalization group methods were invented [13–15] and there have been some analytic studies [2, 3, 6, 8, 9] though it is not easy (almost impossible) to know to what extent the approximate renormalization group methods are precise compared with the real systems. The present author recently proved that 4D non-Abelian lattice gauge theories always exhibit quark confinements (in the sense of Wilson) within the Migdal-Kadanoff approximate renormalization group methods that are believed to be rather accurate as the first approximation [13, 15, 10, 11], see also [16, 17].

In the case of the Heisenberg model in which s_x, the spin variable at the lattice point $x \in Z^2$, takes its values on $S^2 = \{(s^1, s^2, s^3) \in R^3;\ \|s\| = 1\}$, the methods developed in [11] (see also [12]) do not work, and we need new tricks. These new tricks together with the methods in [11] enable us to establish:

Theorem. *The* 2D *Heisenberg model is always in the massive phase within the Migdal-Kadanoff approximate renormalization group methods.*