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The High-Temperature Phase of Long-Range Spin Glasses

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Abstract. We analyze the high-temperature phase of long-range Ising- and N-vector spin glasses with exchange couplings $\{J_{ij}\}$, $i, j \in Z^d$, which are independent random variables with $\bar{J}_{ij}=0$ and $|\overline{J^p_{ij}}| \leqq \gamma^p p! |i-j|^{-p\alpha d}$, $p=2,3,\ldots,\gamma$ is a finite constant and $\alpha > \frac{1}{2}$. We show that, for sufficiently high temperatures, the equilibrium state in the thermodynamic limit is (weakly) unique, and the quenched average of the square of connected correlations $\langle \sigma_A; \sigma_B \rangle_\beta$ decays like $\mathrm{dist}(A, B)^{-2\alpha d}$, despite of Griffiths singularities and the non-summable range of J_{ij} (for $\frac{1}{2} < \alpha \leqq 1$).

1. Introduction: Problems, Notation, Main Results

1.1. Description of the Problems

Real spin glasses are alloys of magnetic and non-magnetic, conducting materials, like iron and gold, manganese and copper. The magnetic atoms or ions (iron) are impurities in a non-magnetic material (gold). The magnetic properties of such a substance are approximately described by a classical spin system with long-range Ruderman-Kittel exchange interactions. We propose to analyze the behaviour of spin correlations in such systems at high temperatures.

Our methods are based on a sequence of high-temperature expansion steps followed by suitable upper bounds on the result of an individual expansion step. These upper bounds simplify the result of an expansion step and reduce the number of terms generated at the next expansion step. They are valid only on the real temperature axis, i.e. our methods only yield convergent bounds for sufficiently high, *real* temperatures, but divergent ones off the real axis. This feature permits us to avoid problems with Griffiths singularities which make full-fledged high-temperature expansions diverge for all or, at least, for very high temperatures, long before a transition temperature is approached.

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