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Ergodic and Topological Properties of Coulombic Periodic Potentials

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Abstract. The motion of a classical pointlike particle in a two-dimensional periodic potential with negative coulombic singularities is examined. This motion is shown to be Bernoullian for many potentials and high enough energies. Then the motion on the plane is a diffusion process. All such motions are topologically conjugate and the periodic orbits can be analysed with the help of a group.

1. Introduction

As Markus and Meyer showed in [18], neither integrability nor ergodicity are generic properties of Hamiltonian systems with two degrees of freedom. Nevertheless, these extreme cases are much better understood than the complicated mixture inbetween. But, according to Berry "No smooth Hamiltonian of the type $H = \text{kinetic} + \text{potential}$ has been proven to be ergodic [7]." Though it is well-known (see e.g. Anosov [2]) that the geodesic flow on a compact surface of strictly negative curvature *is* ergodic, this example is considered unphysical, since Efimov [11] showed that no such surface can be isometrically embedded in Euclidean 3-space.

One purpose of this paper is to show how such surfaces arise in the analysis of two-dimensional periodic potentials with negative coulombic singularities. This is the content of Sect. 2.

In Sect. 3 we give a new regularisation scheme for the collision orbits, which is well adapted to our purposes.

The measure-theoretical properties of the Hamiltonian motion are explored in Sect. 4. For a large class of Coulombic potentials the flow is shown to be Bernoullian. An account of the impacts of this notion is given by Ornstein [21]. We recall that another physical example of a Bernoulli system is given by the motion of a billard in a periodic array of convex obstacles, as shown by Gallavotti and Ornstein [13, 14].

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