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Partial Regularity of a Generalized Solution to the Navier-Stokes Equations in Exterior Domain

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Abstract. In this note, we prove two regularity theorems for solutions to the Navier-Stokes equations of an I.B.V.P. in exterior domains. Namely, we prove that the set S of the singular points of a solution, if not empty, has at most 1-Hausdorff measure $H^1(S)=0$. Moreover, the set S is enclosed in a sphere of ray R for any $t>0$. These results are obtained as corollaries to the partial regularity results furnished in [2].

1. Introduction

In this note we study the regularity of suitable solutions to the initial boundary value problem for the nonstationary three-dimensional Navier-Stokes equations in exterior domains. The existence of global-in-time solutions has been proved long ago by Leray and Hopf [12, 14, 15]. They furnish weak solutions of initial boundary value problems. As is well known, two remarkable questions are open about these solutions: the former concerns their uniqueness, the latter their regularity. In this connection, we observe that so far, the regularity of a solution seems to be independent of the regularity of initial data, and recently Scheffer in [27] has proposed a conjecture, namely, that a solution to Navier-Stokes equations exists having an internal singularity, at least in the class of weak solutions verifying only a "generalized energy inequality" as an a priori estimate.

A first result concerning the regularity of weak solutions to the Cauchy problem is due to Leray [15] with his famous "théorème de structure." Later on, the "théorème de structure" was extended to the case of initial boundary value problems [10, 11, 19]. However, we must notice that, so far, in the case of unbounded domains the "théorème de structure" can be obtained for Leray's weak solutions, [8], while for Hopf's weak solutions it is not known, and, on the other hand, we do not know whether Leray's and Hopf's solutions are the same. In [21–26] Scheffer commences and develops an analysis of the set of the possible singular points of a weak solution to the Navier-Stokes equations. Following Caffarelli, Kohn, and Nirenberg [2], a point will be called "singular" for a solution