Commun. Math. Phys. 110, 51–74 (1987)

Communications in
Mathematical
Physics

Asymptotic Completeness and Multiparticle Structure in Field Theories

D. Iagolnitzer[1] and J. Magnen[2]

[1] Service de Physique Théorique, CEN Saclay, F-91191 Gif-sur-Yvette Cedex, France
[2] Centre de Physique Théorique, Ecole Polytechnique, F-91128 Palaiseau Cedex, France

Abstract. Previous proofs of asymptotic completeness and related results on scattering in field theories are restricted to $P(\varphi)_2$ models in the 2- and 3-particle regions. In this paper, new cluster expansions that are well adapted to more direct proofs and generalizations of these results are presented. In contrast to previous ones, they are designed to provide direct graphical definitions of general irreducible kernels satisfying structure equations recently proposed and shown to be closely linked with asymptotic completeness and with the multiparticle structure of Green functions and collision amplitudes in general energy regions. The method can be applied as previously to $P(\varphi)_2$ and can also be extended to theories involving renormalization which are controlled by phase-space analysis. It is here illustrated in detail for the Bethe-Salpeter kernel in φ_2^4, in which case a new proof of its 4-particle decay (which yields asymptotic completeness in the 2-particle region) is given.

1. Introduction

1.1. Background and General Ideas

Past results on asymptotic completeness and related results in constructive theory apply to weakly coupled, superrenormalizable $P(\varphi)_2$ models in the 2-particle [1, 2] and 3-particle [3] region. (For some previous preliminary results related to spectrum see also [0].) In [1 a] the Bethe-Salpeter kernel G is defined through the B.S. equation

$$F = G + F \circ G, \tag{1}$$

where F is the connected, amputated 4-point function and $F \circ G$ is a Feynman-type convolution integral ${}^1_2{=}\,(F)\,{=}\,(G)\,{=}{}^3_4$ with 2-point functions on internal lines. G is then shown to satisfy in Euclidean space-time 3-particle decay, or in an even theory (e.g. φ_2^4) 4-particle decay of the form $e^{-m(1-\varepsilon)d(x_1,\ldots,x_4)}$ with:

$$d(x_1,\ldots,x_4) = 4\left|\frac{x_{1,0}+x_{2,0}}{2} - \frac{x_{3,0}+x_{4,0}}{2}\right| + |x_{1,0}-x_{2,0}| + |x_{3,0}-x_{4,0}|, \tag{2}$$