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Adiabatic Theorems and Applications to the Quantum Hall Effect

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Abstract. We study an adiabatic evolution that approximates the physical dynamics and describes a natural parallel transport in spectral subspaces. Using this we prove two folk theorems about the adiabatic limit of quantum mechanics: 1. For slow time variation of the Hamiltonian, the time evolution reduces to spectral subspaces bordered by gaps. 2. The eventual tunneling out of such spectral subspaces is smaller than any inverse power of the time scale if the Hamiltonian varies infinitly smoothly over a finite interval. Except for the existence of gaps, no assumptions are made on the nature of the spectrum. We apply these results to charge transport in quantum Hall Hamiltonians and prove that the flux averaged charge transport is an integer in the adiabatic limit.

1. Introduction

The adiabatic limit is concerned with the dynamics generated by Hamiltonians that vary slowly in time: $H(t/\tau)$ in the limit that the time scale τ goes to infinity. Quantum adiabatic theorems reduce certain questions about such dynamics to the spectral analysis of a family of operators, and in particular describe the way in which the dynamics tends to follows spectral subspaces of $H(s)$. $s=t/\tau$ is the scaled time. A folk adiabatic theorem states that if $H(s)$ has energy bands bordered by gaps, as in Fig. 1.1, then a system started at $t=0$ in a state corresponding to an energy band bordered by gaps of $H(0)$, will at time t, be in a state of the corresponding energy band of $H(s)$. If, in addition $H(s)$ is infinitely differentiable and s varies over a finite interval, a second folk theorem states that asymptotically the tunneling out of such an energy band is exponentially small in τ. (For Hamiltonians that are k times differentiable, tunneling is bounded by a power of $1/\tau$.) Both folk theorems emphasize the importance of the gap condition and both make no assumptions on the nature of the spectrum in the energy bands. In contrast, most of the proofs of quantum adiabatic theorems do make additional spectral assumptions. Indeed, the earliest results were obtained for finite dimensional, self-adjoint and non-degenerate matrices. In the general case there