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On the Relation Between Classical and Quantum Statistical Mechanics

W. Wreszinski* and G. Scharf

Institut für Theoretische Physik der Universität Zürich, Schoenberggasse 9, CH-8001 Zürich, Switzerland

Abstract. Classical and quantum statistical mechanics are compared in the high temperature limit $\beta = 1/kT \to 0$. While this limit is rather trivial for spin systems, we obtain some rigorous results which suggest (and sometimes prove) different asymptotics for continuous systems, depending on the behaviour of the two-body potential for small distances: the difference between suitable classical and quantum variables vanishes as β^2 for smooth potentials and as $\sqrt{\beta}$ for potentials with hard cores.

I. Introduction

Although nature is governed by quantum mechanics, there is still much interest in classical statistical mechanics. This comes from the belief that for not too low temperatures the structural differences between the two are small. In fact, several deep similarities between classical and quantum *lattice* systems with regard to phase transitions are apparent [1]. Rigorous results on the low temperature behaviour of even the simplest quantum lattice systems from the point of view of ground state properties, namely, ferromagnetic quantum spin systems, are as yet incomplete [2]. For quantum continuous systems the situation is much worse, at least concerning rigorous results: very little is known on phase transitions for interacting systems, and low temperature properties are studied for just a few one-dimensional models.

Another reason for interest in classical statistics is the fact that it is an approximation to quantum statistics in a precise sense. There is an extensive literature on the classical $\hbar \to 0$ limit of quantum statistics (see [3–5] and references given there). It is a weakness of these studies that no correction terms are provided. Then the limit $\hbar \to 0$ remains mathematical, because physically $\hbar$ cannot be varied. A reasonable statement for continuous systems would be: the difference between

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* Supported in part by FAPESP. Permanent address: Instituto de Fisica, Universidade de Sao Paulo, Sao Paulo, Brazil