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On Charge Conjugation

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Abstract. The group of automorphisms of the conformal algebra su(2, 2) has four components giving the usual four components of symmetries of space time. Only two of these components extend to symmetries of the conformal superalgebra – the identity component and the component which induces the parity transformation, P, on space time. There is no automorphism of the conformal superalgebra which induces T or PT on space time. Automorphisms of su(2, 2) which belong to these last two components induce transformations on the conformal superalgebra which reverse the sign of the odd brackets. In this sense conformal supersymmetry prefers CP to CPT. The operator of charge conjugation acting on spinors, as is found in the standard texts, induces conformal inversion and hence a parity transformation on space time, when considered as acting on the odd generators of the conformal superalgebra. Although it commutes with Lorentz transformations, it does not commute with all of su(2, 2). We propose a different operator for charge conjugation. Geometrically it is induced by the Hodge star operator acting on twistor space. Under the known realization of conformal states from the inclusion SU(2, 2) $\rightarrow$Sp(8) and the metaplectic representations, its action on states is induced by the unique (up to phase) antilinear intertwining operator between the two metaplectic representations. It is consistent with the split orthosymplectic algebras and hence, by the inclusion of the superconformal in the orthosymplectic, with the orthosymplectic algebra.

The conformal superalgebra of Minkowski space-time is a special case of a class of superalgebras defined in [12]. We shall give a definition of a subclass of these superalgebras in Sect. 2, and will be interested in studying their automorphisms. We begin with some notational preliminaries.

Let V be a complex vector space endowed with a (pseudo) Hermitian scalar product (,). That is, (,) assigns a complex number (u, v) to a pair of vectors u and v in V and satisfies

$$(au_1 + bu_2, v) = a(u_1, v) + b(u, v) \qquad \text{linearity in } u\,,$$

$$(v, u) = (\overline{u, v}) \qquad \text{Hermitian property}\,,$$