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A Mathematical Theory of Gravitational Collapse

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Abstract. We study the asymptotic behaviour, as the retarded time u tends to infinity, of the solutions of Einstein's equations in the spherically symmetric case with a massless scalar field as the material model. We prove that when the final Bondi mass M_1 is different from zero, as $u \to \infty$, a black hole forms of mass M_1 surrounded by vacuum. We find the rate of decay of the metric functions and the behaviour of the scalar field on the horizon.

0. Introduction

In [1] we began the study of the global initial value problem for Einstein's equations $R_{\mu\nu} = 8\pi\partial_\mu\phi\partial_\nu\phi$ in the spherically symmetric case with a massless scalar field ϕ as the material model. Using a retarded time coordinate r, the spacetime metric can be put in the form

$$ds^2 = -e^{2\nu}du^2 - 2e^{\nu+\lambda}dudr + r^2 d\Sigma^2,$$

where $d\Sigma^2$ is the metric of the standard 2-sphere. The problem is formulated most simply in terms of the function $h := \partial(r\phi)/\partial r$. We define

$$g := \exp\left[-4\pi \int_r^\infty (h-\bar{h})^2 \frac{dr}{r}\right], \qquad D := \frac{\partial}{\partial u} - \frac{1}{2}\bar{g}\frac{\partial}{\partial r},$$

and

$$m := \frac{r}{2}\left(1 - \frac{\bar{g}}{g}\right), \qquad \xi := 2rD\bar{h},$$

where, if f is a function of u and r, we denote by $\bar{f}$ the mean value function of f:

$$\bar{f}(u,r) := \frac{1}{r}\int_0^r f(u,r')dr'.$$

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